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Optimal Design of Uncertain Systems Under Stochastic Excitation

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The structural synthesis problem of uncertain linear systems subjected to uncertain loads is considered. Uncertain system parameters are modeled as random variables with a prescribed joint probability density function, whereas the loads are modeled as stochastic processes. Second-order probabilistic descriptors are combined with approximate extreme response theories to obtain reliability estimates for the systems. A multicriterion optimal design methodology, based on a preference aggregation rule, is used in this formulation. Optimization is carried out by generating and solving a sequence of explicit approximate problems. The uncertainty in system parameters is taken explicitly in the analysis, and its effect is investigated on the optimal design. It is shown that uncertainties are important because they can change the system reliability significantly. Therefore, in these cases, the effect of uncertainty in the model parameters must be considered explicitly during the design process. An example problem is presented to illustrate the performance of the proposed methodology.

Nomenclature

$\{b\}$	=	vector of system parameters
$[C]$	=	damping matrix
$E_t(\cdot)$	=	expectation operation in time domain
$\{f\}$	=	vector of performance parameters
$g(\cdot)$	=	probability density function
$g_j(\cdot)$	=	response quantity
$[K]$	=	stiffness matrix
$[M]$	=	mass matrix
$n(\cdot)$	=	stationary Gaussian white noise process
$\{P_R\}$	=	vector of structural reliabilities
$\{p\}$	=	compatibility vector
$Q_{ij}(\cdot)$	=	modal cross covariances
$\{q(\cdot)\}$	=	state-space vector
$R(\cdot)$	=	generic design criterion
$r(\cdot)$	=	response quantity
S_i	=	modal energy
S_0	=	power spectral density value
T_i	=	modal energy
t	=	time variable
U_i	=	modal energy
$\{u(\cdot)\}$, $\{\dot{u}(\cdot)\}$,	=	vectors of dynamic displacement, velocity,
$\{\ddot{u}(\cdot)\}$	=	and acceleration, respectively
$w(\cdot)$	=	importance sampling density
$\{x\}$	=	vector of intermediate variables
$\{y\}$	=	vector of design variables
$\eta_i(\cdot)$	=	modal participation coefficient
η_j	=	threshold level
λ_i	=	eigenvalue
$\mu(\cdot)$	=	preference function
$\nu^+(\cdot)$	=	expected rate of up crossing a threshold level
$\sigma^2_{g_j}$, $\sigma^2_{\dot{g}_j}$	=	second-order statistics
$\Phi(\cdot)$	=	Gaussian distribution function
$\{\phi\}_i$	=	right eigenvector
$\{\phi\}_{pi}$	=	position part of the right eigenvector
$\{\chi\}_i$	=	left eigenvector
$\{\chi\}_{pi}$	=	position part of the left eigenvector
~	=	approximate quantity

Introduction

TRADITIONALLY, all design data in structural optimization problems are treated in a deterministic manner. The problem with this approach is that in many cases deterministic optimization gives designs with higher failure probability than unoptimized structures. Therefore, because uncertainties are always present in the design of engineering structures, it is necessary to consider their effect on the optimization process to achieve a balance between cost and safety for the optimal design.[1–3] It is clear that, when a structure is being designed, the environmental loads that the built structure will experience in its lifetime are highly uncertain. The uncertain load time history needed in the dynamic analysis of a structure subjected to environmental loads such as earthquake, wind, water wave excitation, and aerodynamic turbulence is an uncertain value function, and it is best modeled by a stochastic process.[4,5] If the structural parameters are known precisely, then the system response and system reliability can be calculated using well-known techniques from random vibration theory. In the more realistic case, the values of the structural parameters are uncertain. These uncertainties, which result from the numerous assumptions made when modeling the geometry, the boundary conditions, constitutive behavior of the materials involved, etc., can have a significant effect on the behavior of the structure. Probabilistic methods provide the means for incorporating system uncertainties in the analysis by describing the uncertainties as random variables with a prescribed joint probability density function. System responses and system reliabilities that account for the uncertainties in the system parameters are given by the total probability theorem as particular integrals over all of the uncertain parameters. Exact analytical solutions for these multidimensional integrals can only be found for a very limited number of simple systems. For more realistic systems, Monte Carlo simulation[6] or important sampling techniques[7] can be used to provide accurate results for evaluating unconditional system responses. Other methods that have been developed to provide computational tools for approximating the structural responses and reliabilities of uncertain systems subjected to stochastic loads are first-order reliability methods[8] and second-order reliability methods.[9] These methods have been tested for a variety of structural problems, including simple linear and nonlinear systems and primary–secondary systems.[10,11] Recently, a new technique based on Laplace's method for asymptotic approximation of integrals has been developed for evaluating the type of probability integrals encountered in the reliability analysis of uncertain systems subjected to stochastic excitation.[12]

In practical optimization problems, usually more than one objective is required to be optimized. This makes it necessary to formulate the design problem as a multi-objective optimization problem

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and look for a set of compromise solutions. Generally, the approach in multi-objective optimization is to transform the original problem into a scalar problem, which contains the influence of all objectives.[13,14] In the present paper, the objective functions, constrained system responses, and constraints directly imposed on the design variables are treated as design criteria characterized by a range of values and a preference function that describes the preference of using a particular value within the range.[15–17] Values of preference functions lie in the unit interval [0, 1], and they quantify the preference of the user for the various values of the design criteria. A larger preference value for one parameter value compared with another implies that the user prefers the first parameter value more than the other value. By treating the design criteria in this way, the degree to which they are satisfied can be traded off against other design criteria during the optimization process. In this way, the problem is placed into a more flexible and natural framework, where the knowledge of the desired characteristics of the optimal design can be exploited. Once the preference functions for each design criterion have been defined, an overall design evaluation measure is obtained by a preference aggregation rule.[18] This measure is then used as the objective function of the optimization process. Optimal solutions provided by this approach belong to the multi-objective Pareto optimal set.[16]

In general, evaluating design criteria preference functions requires the evaluation of system responses that are nonlinear implicit functions of the actual design variables and uncertain system parameters, and they are available only in a numerical sense (for example, using a finite element method). Thus, to use this approach in structural design problems, numerically efficient algorithms are required to evaluate system responses. One of the objectives in this work is to extend the approximation concepts method[19,20] to the case in which the system is subjected to stochastic loads and some system parameters are uncertain. In this approach, the solution of the original implicit optimization problem is replaced by the solution of a sequence of explicit approximate problems. These approximate problems are generated by constructing high-quality approximations for system responses. Intermediate response quantities and intermediate variables are used to enhance the quality of the approximations. The formulation to be presented represents a generalization of the methodology proposed in Ref. 21 to the case of multi-objective optimization. The paper presents a general methodology for computer-aided multicriteria optimal design that allows all of the different design criteria to be traded off while accounting for modeling and loading uncertainties.

First, the structural optimization problem with reliability constraints is formulated for the case of known system parameters. Reliability constraints are evaluated and written in terms of the solution of a general linear structural system for a class of stochastic excitation. Next, a first-order optimization technique is introduced to solve the optimization problem resulting from the proposed formulation. Approximation concepts are used for an efficient implementation of the methodology. Finally, the method is extended to consider the case of unknown system parameters and their effect on the optimization process.

Formulation

The design decision making process is an iterative procedure where a preliminary design is cycled through different stages to achieve a design that is optimum in some chosen sense. The formulation starts with a preliminary description of the design problem. This includes the physical configuration, all possible loading cases that the structure might experience during its lifetime, the design variables, and the design criteria. Let the vectors $\{y\}$ (y_i, $i = 1, \ldots, N$) and $\{b\}$ (b_i, $i = 1, \ldots, L$) represent the vector of design variables and system parameters, respectively. The design variables are the parameters of the design that are selected to be varied during the optimization process, whereas the system parameters are quantities that may have associated uncertainties. The case of fixed and known system parameters $\{b\}$ is first considered. The general case of uncertain system parameters will be considered later. The performance parameters represent quantities related to the performance of the design. They can take the form of conventional structural parameters such as stresses, displacements, material cost, etc., $\{f\}$ (f_i, $i = 1, \ldots, K$), or other parameters such as structural reliabilities $\{P_R\}$ (P_{R_j}, $j = 1, \ldots, M$). Obviously, these performance parameters are functions of the current vector of design variables $\{y\}$ and the fixed vector of system parameters $\{b\}$. The functional requirements or performance specifications for the performance parameters constitute the design criteria during the design process.

In this formulation, a multicriterion optimal design methodology based on preference aggregation rules is used to transform the multicriteria optimization problem to a single objective problem. A proposed design is first evaluated on the basis of each design criterion, and then this information is aggregated into a single design evaluation measure. Each design criterion is evaluated by a preference function, which quantifies the preference of the designer for the various values of the design criterion involved in the problem. The overall single design evaluation measure is computed using a preference aggregation rule, which is a functional relationship between a single design evaluation measure and the individual preference values for all of the design criteria. This single measure lies in the unit interval, and the optimal design is the one that maximizes such a measure. In this paper, a preference aggregation rule based on the method of the minimum operator[16,18] is used to transform the multicriteria optimization problem to a single objective problem. The overall design evaluation measure is defined as

$$\mu(\{y\}) = \min\Big[\mu_{f_i}(\{y\})\;\; i \in I_f,\;\; \mu_{P_{R_j}}(\{y\})\;\; j \in I_p,\;\; \mu_{y_k}(\{y\})\;\; k \in I_y\Big] \tag{1}$$

where $\mu(\{y\})$ is the overall design evaluation measure for the design $\{y\}$, $I_f = \{1, \ldots, K\}$, $I_p = \{1, \ldots, M\}$, $I_y = \{1, \ldots, N\}$, $\mu_{f_i}(\{y\})$, $i \in I_f$, are the preference functions of conventional performance parameters for the design $\{y\}$, $\mu_{P_{R_j}}(\{y\})$, $j \in I_p$, are the preference functions of reliability performance parameters for the design $\{y\}$, and $\mu_{y_k}(\{y\})$, $k \in I_y$, are the preference functions for the design variables.

The maximum degree of overall satisfaction can be achieved by maximizing the overall design evaluation measure μ. Then the problem becomes

$$\max\min\Big[\mu_{f_i}(\{y\})\;\; i \in I_f,\;\; \mu_{P_{R_j}}(\{y\})\;\; j \in I_p,\;\; \mu_{y_k}(\{y\})\;\; k \in I_y\Big] \tag{2}$$

From this formulation, it is clear that any constraints imposed directly on the design variables are treated as additional design criteria. Note that an alternative interpretation of problem (2) is in the context of fuzzy optimization. In that case, the preference functions are viewed as membership functions for the fuzzy sets of acceptable performances of the design criteria. The choice of the shape of these preference functions for the design criteria can have an impact on the overall design process. In practice, piecewise linear preference functions are commonly used because of their simplicity and expediency. In particular, trapezoidal preference functions are generally used for constrained system responses and for constraints directly imposed on the design variables. Note, however, that other shapes for preference functions such as concave and convex can also be used in this formulation. These nonlinear shapes offer potential benefits in terms of realism, and they can be chosen consistent with varying perception of the decision maker or designer. Noted also that the definition of the overall design evaluation measure given in Eq. (1) can be easily modified to consider different degrees of importance for the design criteria. In that case, importance weights can be assigned to each of the design criterion.[15,16]

Reliability Computations

In this study, attention is directed toward problems in which the stochastic excitation is a stationary Gaussian white noise process with zero mean. A white noise process is a process whose power spectral density function is constant over the whole spectrum. Because of its mathematical simplicity, it is often used as an approximation to a great number of physical phenomena. It is well known, from classical random vibration theory, that displacement and velocity responses of a linear system subjected to a Gaussian excitation are Gaussian processes. Therefore, any response quantity $g_j(t, \{y\})$ that is linearly dependent on the displacement and velocity is also a Gaussian process. In reliability-based structural optimization requirements are specified for the reliability of the structural components and/or the total system. In this formulation, the constraints are related to single failure modes, where failure is assumed to occur when the response $g_j(t, \{y\})$ reaches some critical level η_j for the first time in the stationary portion of the response. The probability that $g_j(t, \{y\})$ has not reached the level η_j prior to time t can be obtained using available results from random vibration theory. These results are based on the expected rate of up crossing and down crossing through levels η_j and $-\eta_j$, respectively. The expected rate of up crossing a given level η_j, $\nu^+(\{y\})$, is given in terms of the steady-state second-order statistics $\sigma^2_{g_j} = E_t[g^2_j(t, \{y\})]$ and $\sigma^2_{\dot{g}_j} = E_t[\dot{g}^2_j(t, \{y\})]$ as[5]

$$\nu^+(\{y\}) = \left(\sigma_{\dot{g}_j}/2\pi\sigma_{g_j}\right)\exp\left[-\eta_j^2/2\sigma^2_{g_j}\right] \tag{3}$$

where $E_t(\cdot)$ is the mathematical expectation with respect to uncertainty in the time domain and σ_{g_j} and $\sigma_{\dot{g}_j}$ are the conditional standard deviation of the response $g_j(t, \{y\})$ and its time derivative, respectively. The reliability function for the design $\{y\}$, $P_{R_j}(\{y\})$, is the probability that $g_j(t, \{y\})$ has not reached the value η_j throughout the interval $[0, t]$. For a high-threshold level η_j, it can be assumed that the events of crossing such a level occur independently according to a Poisson process with mean rate $\nu^+(\{y\})$, in which case the reliability function $P_{R_j}(\{y\})$ can be approximated by[4]

$$P_{R_j}(\{y\}) = \mathrm{prob}\left[\left\{\max_{[0,t]} g_j(t, \{y\})\right\} \le \eta_j\right]$$
$$\approx \exp(-2\nu^+(\{y\})t), \qquad j = 1, \ldots, M \tag{4}$$

where prob[·] is the probability that the expression in parenthesis is true and $P_{R_j}(\{y\}) = 1$ at $t = 0$ for any $\eta_j > 0$ and any design $\{y\}$.

Response Statistics

The steady-state second-order statistics σ_{g_j} and $\sigma_{\dot{g}_j}$ of the response functions $g_j(\cdot)$, $j = 1, \ldots, M$, are now written in terms of the solution of a general underdamped linear system. The solution of the system is obtained by modal analysis. The modal response is then used to derive the second-order statistics of the vector of state-space variables, which are then used to compute the system response statistics σ_{g_j} and $\sigma_{\dot{g}_j}$.

Modal Approach

The equation of motion of an n-degree-of-freedom linear structural system subjected to external forces can be cast in the form

$$[M]\{\ddot{u}(t)\} + [C]\{\dot{u}(t)\} + [K]\{u(t)\} = \{p\}n(t) \tag{5}$$

where $\{u(t)\}$ is the displacement response vector of dimension n; $[M]$, $[C]$, and $[K]$ are the mass, damping, and stiffness matrices of dimension $n \times n$; $\{p\}$ is a vector of dimension n relating the force to the degrees of freedom of the system; and $n(t)$ is stationary Gaussian white noise with zero mean and constant power spectral density S_0. In general, the matrices $[M]$, $[C]$, and $[K]$ depend on the vector of design variables $\{y\}$ and fixed system parameters $\{b\}$. Therefore, the displacement response vector $\{u(t)\}$ is also a function of $\{y\}$ and $\{b\}$.

To consider the general case of nonclassically damped structures, the equation of motion (5) is recast into the $2n$ state-space form

$$[M^*]\{\dot{q}(t)\} + [K^*]\{q(t)\} = \{p(t)\}^* \tag{6}$$

where

$$\{q(t)\} = \begin{Bmatrix}\{\dot{u}(t)\}\\ \{u(t)\}\end{Bmatrix} \tag{7}$$

is the vector of state-space variables and

$$[M^*] = \begin{pmatrix}[0] & [M]\\ [M] & [C]\end{pmatrix}, \qquad [K^*] = \begin{pmatrix}-[M] & [0]\\ [0] & [K]\end{pmatrix}$$
$$\{p(t)\}^* = \begin{Bmatrix}\{0\}\\ \{p\}n(t)\end{Bmatrix} \tag{8}$$

The solution of Eq. (6) is carried out by standard modal analysis. In this approach, the solution is represented as a linear combination of complex mode shapes of the form

$$\{q(t)\} = \sum_{i=1}^{2n}\{\phi\}_i\eta_i(t) \tag{9}$$

where $\eta_i(t)$, $i = 1, \ldots, 2n$, are the modal participation coefficients, and $\{\phi_i\}$, $r = 1, \ldots, 2n$, are the complex right eigenvectors corresponding to problem (6). The modal coordinate η_i satisfies a complex modal equation, which can be expressed as[22]

$$\dot{\eta}_i(t) - \lambda_i\eta_i(t) = \frac{\{\chi\}^t_{pi}\{p\}}{(2\lambda_i T_i + S_i)}n(t), \qquad i = 1, \ldots, 2n \tag{10}$$

where λ_i is the ith eigenvalue of problem (6), which can be expressed in terms of the modal energies T_i, S_i, and U_i as

$$\lambda_i^2 T_i + \lambda_i S_i + U_i = 0 \tag{11}$$

where the modal energies are defined as

$$T_i = \{\chi\}^t_{pi}[M]\{\phi\}_{pi}, \qquad U_i = \{\chi\}^t_{pi}[K]\{\phi\}_{pi}$$
$$S_i = \{\chi\}^t_{pi}[C]\{\phi\}_{pi} \tag{12}$$

and where $\{\phi\}_{pi}$ and $\{\chi\}_{pi}$ are the position parts (the last n components) of the right and left eigenvectors of problem (6), respectively, and all other terms have already been defined. For underdamped systems, the modes appear in complex conjugate pairs, so if they are arranged such that $\lambda_{n+i} = \bar{\lambda}_i$, $\{\phi\}_{n+i} = \{\bar{\phi}\}_i$, and $\{\chi\}_{n+i} = \{\bar{\chi}\}_i$, $i = 1, \ldots, n$, where $\bar{\lambda}_i$, $\{\bar{\phi}\}_i$, and $\{\bar{\chi}\}_i$ are the complex conjugates of λ_i, $\{\phi\}_i$, and $\{\chi\}_i$, respectively, then the modal participation coefficients appear in complex conjugate pairs, that is, $\eta_{n+i}(t) = \bar{\eta}_i(t)$, $i = 1, \ldots, n$.

Second-Order Statistics

As mentioned, a stationary zero mean Gaussian stochastic process is used to model the uncertainty of the external force. The state vector $\{q(t)\}$ is also a zero mean Gaussian process due to the linearity of the system, and it is fully described by the covariance matrix $E_t(\{q(t)\}\ \{q(t+\tau\}^t)$ for any time lag τ (Ref. 5). Of particular importance are the zero-lag second-order statistics because they provide useful information about the level of system responses and they can also be used to compute mean rates of level crossing for reliability estimation. The covariance matrix of the state vector $\{q(t)\}$ at zero lag can be written in terms of the modal cross covariances as

$$E_t(\{q(t)\}\quad\{q(t)\}^t) = \sum_{i=1}^{2n}\sum_{j=1}^{2n}\{\phi\}_i\{\phi\}^t_j E_t[\eta_i(t)\quad\eta_j(t)] \tag{13}$$

where $E_t[\eta_i(t)\ \ \eta_j(t)]$ is the cross covariance at zero lag for the modal participation coefficients. Assuming that the excitation function $n(t)$ is a zero mean Gaussian white noise process, it can be shown that the steady-state cross covariance $Q_{ij}(t) = E_t[\eta_i(t)\ \ \eta_j(t)]$ satisfies[4,21]

$$(\lambda_i + \lambda_j)Q_{ij}(t) = -2S_0\left[\frac{\{\chi\}^t_{pi}\{p\}}{(2\lambda_i T_i + S_i)}\frac{\{\chi\}^t_{pj}\{p\}}{(2\lambda_j T_j + S_j)}\right]$$
$$i, j = 1, \ldots, 2n \tag{14}$$

Equation (14) specifies the components of the steady-state covariance matrix of the modal participation coefficients. Because the mathematical model (6) depends on the vector of design variables $\{y\}$ and fixed system parameters $\{b\}$, it is clear that $Q_{ij}(t)$ is also a function of these two vectors.

Response Quantities

The response quantities of interest are given as a linear combination of the displacement vector $\{u(t)\}$, that is,

$$r(t) = \{\beta\}^t\{u(t)\} \tag{15}$$

where $r(t)$ represents a response quantity, that is, $g_j(t)$, and $\{\beta\}$ a vector of constant components. Note that, in general, the vector $\{\beta\}$ depends on the vector of design variables and system parameters. This dependence arises when, for example, the response quantity $r(t)$ is a stress or force member and some of the cross-sectional properties (dimensions or mechanical properties) of the member are design variables or system parameters. In this case, cross-sectional properties are involved in the computation of the force member. The stationary second-order statistics of the response, which are needed to estimate the system reliability, can be computed directly from Eq. (13). For example, the variance σ_r^2 of the response $r(t)$ is given by

$$\sigma_r^2 = E_t[r^2(t)] = \{\beta\}^t \sum_{i=1}^{2n} \sum_{j=1}^{2n} \{\phi\}_{pi} \{\phi\}_{pj}^t Q_{ij}(t) \{\beta\} \tag{16}$$

where all terms have been defined. The variance of the response time derivative $\sigma_{\dot{r}}^2$ is computed in a similar manner. Because of the symmetry of $Q_{ij}(t)$, that is, $Q_{ij}(t) = Q_{ji}(t)$, only $n(2n + 1)$ modal cross-covariance responses are distinct, and therefore, the number of modal components $Q_{ij}(t)$ to be analyzed is significantly reduced. Also, recall that the eigenvalues λ_i, $i = 1, \ldots, 2n$, appear in complex conjugate pairs, that is, $\lambda_{n+i} = \bar{\lambda}_i$, $i = 1, \ldots, n$. Then, it is easy to verify that out of the $n(2n + 1)$ quantities to be considered, n are real, whereas the remaining $2n^2$ appear in complex conjugate pairs. Finally, the interpretation of Eq. (16) as the modal superposition formula for the system response second-order moments can be advantageously used to justify modal truncation techniques that reduce considerably the number of contributing modes in the equation. Utilizing these observations in the numerical implementation of the method reduces significantly the computational effort involved in the analysis.

Optimization Scheme

Once the system response statistics have been computed, the reliability functions can be evaluated directly from Eq. (4). Then, the optimization problem (2) is a general nonlinear deterministic optimization problem. In general, optimization algorithms that use first-order information are considered to be the most efficient schemes for nonlinear optimization problems. In this paper, a classical first-order gradient-based optimizer has been selected for the solution of the optimization problems that arise from the proposed formulation. The maximization of the overall design evaluation measure, $\mu(\{y\})$, can be more efficiently implemented by rewriting problem (2) as an equivalent optimization problem of the following form: max λ subject to

$$\begin{aligned} &\lambda \le \mu_{f_i}(\{y\}), && i \in I_f \\ &\lambda \le \mu_{P_{R_j}}(\{y\}), && j \in I_p \\ &\lambda \le \mu_{y_k}(\{y\}), && k \in I_y \end{aligned} \tag{17}$$

The constraints of problem (17) are not differentiable at points at which the preference functions are nondifferentiable. The sources of nondifferentiability for these functions come from the limitations to a range contained in the interval [0,1] and nondifferentiability of the functions themselves. Figure 1 shows a generic plot of a preference function that illustrates this concept. In Fig. 1, y_1 and y_4 are points of nondifferentiability due to the lower bound $\mu \ge 0$, y_2 due to the upper bound $\mu \le 1$, and y_3 is an intrinsic nondifferentiable point due to the shape of the function itself. The first type of nondifferentiable points is avoided by limiting the optimization domain in problem (17) to the effective support D of the design evaluation measure μ, which is given by

$$D = \bigcap_{i \in I_f} \left\{\{y\} \,\middle|\, \mu_{f_i}(\{y\}) > 0\right\} \bigcap_{j \in I_p} \left\{\{y\} \,\middle|\, \mu_{P_{R_j}}(\{y\}) > 0\right\} \bigcap_{k \in I_y} \left\{\{y\} \,\middle|\, \mu_{y_k}(\{y\}) > 0\right\} \tag{18}$$

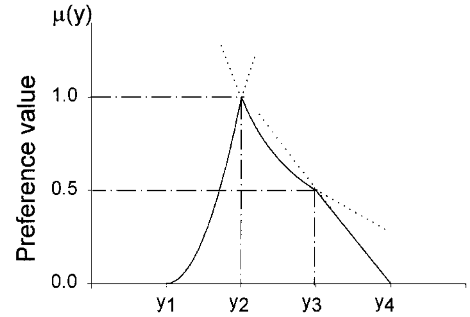

Fig. 1 Generic preference function.

If the feasible domain is limited to the closure of D, denoted by $\bar{D}$, the first type of nondifferentiable points are eliminated from the problem. This constraint is easily included in formulation (17) by appending the side constraint $\lambda \ge 0$. The second type of nondifferentiable points, that is, when $\mu = 1$, can also be eliminated in formulation (17) by extending the function μ above the limit $\mu = 1$ and imposing the additional constraint $\lambda \le 1$. Finally, the third type of nondifferentiability, that is, points where μ is intrinsically nondifferentiable, can be coped with in two ways: extending the corresponding curves beyond their domains of definition (dotted lines in Fig. 1) and considering each branch as a separate preference function in formulation (17), or by smoothing the curve about that point, for example, by a cubic spline interpolant. The first approach is used in this formulation and the optimization problem is written as[16] follows: max λ subject to

$$\begin{aligned} &\lambda \le \bar{\mu}_{f_i}(\{y\}), && i \in I_{\bar{f}} \\ &\lambda \le \bar{\mu}_{P_{R_j}}(\{y\}), && j \in I_{\bar{p}} \\ &\lambda \le \bar{\mu}_{y_k}(\{y\}), k \in I_{\bar{y}}, && 0 \le \lambda \le 1 \end{aligned} \tag{19}$$

where $\bar{\mu}_{f_i}(\{y\})$, $\bar{\mu}_{P_{R_j}}(\{y\})$, and $\bar{\mu}_{y_k}(\{y\})$ are the extended preference functions and $I_{\bar{f}}$, $I_{\bar{p}}$, and $I_{\bar{y}}$ are set of indices for the branches of the original preference functions. In this form, the constraints of problem (19) are differentiable, and therefore, the optimization problem can be solved using a classical first-order gradient-based optimizer, provided that the optimization is initiated from a point $(\{y\}_0) \in \bar{D}$. In this case, the optimal solution of the problem is the one that gives the maximum overall degree of preference with respect to all design criteria.

For the optimization scheme described earlier, it is necessary to initiate the optimization process from a point $(\{y\}_0) \in \bar{D}$. Thus, if such a point is not readily available, that is, the initial design violates some of the constraints, an initial problem (phase I problem) must be solved to find such a point. From the way the preference functions are constructed, it is always straightforward to characterize $\bar{D}$. For example, if the support of the preference function μ_R, of a generic design criterion $R(\{y\})$, is given by the set

$$\bar{D}_{\mu_R} = \{\{y\} \mid R(\{y\}) - R_S \le 0, -R(\{y\}) + R_I \le 0\} \tag{20}$$

where R_I and R_S are the lower and upper bounds of the support of the preference function μ_R, then the set $\bar{D}$ for the problem can always be written as

$$\bar{D} = \{\{y\} \mid h_i(\{y\}) \le 0, i \in I_h\} \tag{21}$$

where h_i, $i \in I_h$, are functions that can be defined directly from the characterization given in Eq. (20) and I_h is a set of indices. With this notation, the phase I problem can be defined as follows: min z subject to

$$z \ge h_i(\{y\}), \qquad i \in I_h \tag{22}$$

For a given design $(\{y\}_0)$, the point $(\{y\}_0, z_0)$, where

$$z_0 = \max_{i \in I_h} h_i(\{y\}_0) \tag{23}$$

is feasible for problem (22). If $(\{y\}^*, z^*)$ is the optimal solution of problem (22), then $\bar{D} \neq \Phi$ (empty set) if and only if $z^* \leq 0$, and $(\{y\}^*)$ can be used as the initial design for the next phase. It has been shown that this optimization scheme is reliable and presents good convergence properties for the solution of optimization problems similar to problem (17) (Ref. 16).

Approximation for Response Quantities

It is clear from Eqs. (15) and (16) that the stationary second-order statistics of the response depend on the modal energies T_i, S_i, and U_i, $(i = 1, \ldots, 2n)$, and the position parts of the right and left eigenvectors $\{\phi\}_i$ and $\{\chi\}_i$, $(i = 1, \ldots, 2n)$, respectively. At the same time, these quantities are implicit nonlinear functions of the vector of design variables $\{y\}$. Therefore, the evaluation of the extended preference functions of the reliability constraints $\bar{\mu}_{P_{R_j}}(\cdot)$, which requires the second order statistics of the response, implies the repeated evaluation of the system response (structural analyses), which can be prohibitively expensive from the numerical point of view during the design process. A similar situation occurs with the evaluation of the extended preference functions of conventional structural parameters $\bar{\mu}_{f_i}(\cdot)$, $i \in I_{\tilde{f}}$. The basic ideas used in the approximation concepts method[19,22] are extended in a straightforward manner for the efficient evaluation of the second-order statistics of the response. In this approach, the complex modal energies T_i, S_i, and U_i are chosen as intermediate response quantities, whereas the position parts of the right and left eigenvectors are assumed to be invariant. The modal energies are approximated by using a convex linearization with respect to selected intermediate design variables $\{x\}$ as

$$\tilde{T}_i = T_{i0} + \sum_{(+)} \frac{\partial T_i(\{x_0\})}{\partial x_j}(x_j - x_{j0}) + \sum_{(-)} \frac{\partial T_i(\{x_0\})}{\partial x_j}\frac{x_{j0}}{x_j}(x_j - x_{j0}) \tag{24}$$

$$\tilde{S}_i = S_{i0} + \sum_{(+)} \frac{\partial S_i(\{x_0\})}{\partial x_j}(x_j - x_{j0}) + \sum_{(-)} \frac{\partial S_i(\{x_0\})}{\partial x_j}\frac{x_{j0}}{x_j}(x_j - x_{j0}) \tag{25}$$

$$\tilde{U}_i = U_{i0} + \sum_{(+)} \frac{\partial U_i(\{x_0\})}{\partial x_j}(x_j - x_{j0}) + \sum_{(-)} \frac{\partial U_i(\{x_0\})}{\partial x_j}\frac{x_{j0}}{x_j}(x_j - x_{j0}) \tag{26}$$

where $T_{i0} = T_i(\{x_0\})$, $S_{i0} = S_i(\{x_0\})$, $U_{i0} = U_i(\{x_0\})$, $\{x_0\} = \{x(\{y_0\})\}$, and $\{y_0\}$ corresponds to the vector of design variables $\{y\}$ when the values of the components are equal to their base design values, $\sum_{(+)}$ means summation over the variables for which $\partial(\cdot)(\{x_0\})/\partial x_j$ is positive and $\sum_{(-)}$ contains the remaining variables. An attractive property of this linearization is that it yields the most conservative approximation among all of the possible combination of direct/reciprocal variables.[23]

The partial derivatives used in the approximations are evaluated assuming that the position parts of the eigenvectors are invariant, that is,

$$\frac{\partial T_i}{\partial x_j} = \{\chi\}_{pi}^t \frac{\partial [M]}{\partial x_j}\{\phi\}_{pi}, \qquad \frac{\partial U_i}{\partial x_j} = \{\chi\}_{pi}^t \frac{\partial [K]}{\partial x_j}\{\phi\}_{pi}$$
$$\frac{\partial S_i}{\partial x_j} = \{\chi\}_{pi}^t \frac{\partial [C]}{\partial x_j}\{\phi\}_{pi} \tag{27}$$

Introducing the earlier approximations in Eq. (14), we obtain an explicit approximation for the modal cross covariances $Q_{ij}(t)$, $i, j = 1, \ldots, 2n$, in terms of $\{y\}$. These approximate covariances are then used to construct approximations for the second-order statistics of the response $\tilde{\sigma}_r$ and $\tilde{\sigma}_{\dot{r}}$. Finally, the approximate steady-state second-order statistics of the response $r(t)$ are used in combination with Eqs. (3) and (4) to estimate the reliability function for the design $\{y\}$. Note that approximation concepts can also be used to approximate conventional performance parameters that are implicit nonlinear functions of the vector of design variables $\{y\}$. In this way, the implicit optimization problem (19) is replaced by an explicit approximate problem, and the solution of the original problem is obtained by the solution of a sequence of explicit suboptimization problems.[24]

System Uncertainties

Response predictions made during the design process are usually based on system models with uncertain parameters because the properties that will be exhibited by the system when completed are not known precisely. In this formulation, the system parameters b_i, $i = 1, \ldots, L$, are modeled using a prescribed joint probability density function $g(\{b\})$. This function indicates the relative plausibility of the possible values of the uncertain parameters in R^L. The probability distribution is always conditional on the information used. If relevant data are available, an updated version of the $g(\{b\})$ can be derived using Bayes theorem. Under uncertain conditions, conventional performance parameters and reliability performance parameters represent conditional quantities for a design $\{y\}$ given the vector of system parameters $\{b\}$. The performance parameters that account for the uncertainties in the system parameters can be obtained by the total probability theorem. For example, the overall or unconditional reliability considering the uncertainties in the system parameters is given by the total probability theorem in the multidimensional form

$$P_{R_j(\mathrm{uncon})}(\{y\}) = \int_{R^L} P_{R_j}(\{y\} \mid \{b\}) g(\{b\}) \mathrm{d}\{b\} \tag{28}$$

where $P_{R_j(\mathrm{uncon})}(\{y\})$ is the unconditional reliability for the design $\{y\}$ and $P_{R_j}(\{y\} \mid \{b\})$ is the conditional reliability for the design $\{y\}$ given the system parameters $\{b\}$. The integral in Eq. (28) represents the total reliability that accounts for the uncertainties in the system parameters, as well as the uncertainties in the loads. Equivalently, the overall failure probability can be written in terms of the conditional failure probability. Note that the unconditional performance parameters are deterministic quantities, and therefore, they can be used directly in the general formulation presented before.

In practice, the multidimensional integral (28) rarely, if ever, can be integrated analytically. On the other hand, numerical integration can be very costly and is usually unaffordable for more than a few variables. Standard simulation methods may also require a very large number of integrand evaluations (structural analyses) to get accurate results. In these cases, efficient importance sampling simulation methods[6,7,25,26] or asymptotic methods[12,27] must be used. Importance sampling techniques can provide accurate estimates of multidimensional probability integrals while substantially reducing the usually large computer effort required in the straightforward Monte Carlo simulation method. To this end, the integral (28) is rewritten in the form

$$P_{R_j(\mathrm{uncon})}(\{y\}) = \int_{R^L} \frac{P_{R_j}(\{y\} \mid \{b\}) g(\{b\})}{w(\{b\})} w(\{b\})\, \mathrm{d}\{b\}$$
$$= \int_{R^L} \varphi(\{y\}, \{b\}) w(\{b\})\, \mathrm{d}\{b\} \tag{29}$$

where $w(\{b\})$ is the importance sampling density. By the use of simulation on Eq. (29), $P_{R_j(\mathrm{uncon})}(\{y\})$ is estimated by the sample mean of $\varphi = P_{R_j} g/w$:

$$P_{R_j(\mathrm{uncon})}(\{y\}) \approx \frac{1}{m}\sum_{k=1}^{m} \varphi\big(\{y\}, \{b\}^{(k)}\big) \tag{30}$$

where m is the number of simulations and each sample $\{b\}^{(k)}$ is drawn from the importance sampling distribution $w(\{b\})$. The choice of $w(\{b\})$ is a critical factor in obtaining an accurate estimate with fewer simulations than those required in standard simulation methods. The basic idea in this technique is to generate most of the samples in the region that contributes significantly to the multidimensional integral, so that the importance sampling simulations will converge rapidly to the value of the probability integral.[28] On the other hand, asymptotic methods use a technique based on Laplace's method for asymptotic approximation of multidimensional integrals. The asymptotic approximation uses an expansion of the logarithm of the integrand about the point that corresponds to the maximum of the integrand of the multidimensional probability integral. The approximation is asymptotically correct, that is, the sharper the peak of the integrand about its maximum value, the more accurate the value of the asymptotic approximation is expected to be. In the case of a finite number of local maxima of the integrand, the asymptotic contributions for each maximum point are added to obtain an asymptotic approximation for the multidimensional probability integral. This result is because the integral can be decomposed into a finite sum of integrals over the disjoint subregions of a partition of R^L, where each subregion contains one and only one maximum point.

Under uncertain system parameters conditions, the performance parameters are approximated in terms of the vector of design variables $\{y\}$ as well as the vector of uncertain system parameters $\{b\}$. In this case, intermediate response quantities are approximated by using a convex linearization with respect to appropriate intermediate variables that are explicit functions of the design variables y_i, $i = 1, \ldots, N$, and uncertain system parameters b_j, $j = 1, \ldots, L$. In this manner, explicit approximate performance parameters in terms of approximate quantities are obtained. The approximate responses are then used in combination with any of the available methodologies for evaluating the general class of multidimensional integrals of the form given by Eq. (28). For example, Monte Carlo simulation methods or importance sampling techniques can be implemented very efficiently because now all performance parameters are explicit functions of the vector of design variables $\{y\}$ and the vector of uncertain system parameters $\{b\}$.

Illustrative Example

The following example problem has been chosen to illustrate the effect of uncertain system parameters on the optimal design of a simple structural system subjected to a stochastic excitation. The problem consists in a five-story shear structure (Fig. 2a) modeled as a five-degree-of-freedom chainlike system of masses connected by equivalent springs k_i, $i = 1, \ldots, 5$ (Fig. 2b). All masses are taken as known and equal to 3.0×10^4 kg. The stiffness of the equivalent springs corresponds to the stiffness of the column elements of the system, which have square cross sections with an elastic modulus $E = 2.0 \times 10^9$ N/m^2. To complete the formulation of the system, some amount of damping is added to the model. The system is subjected to a base acceleration modeled as a stationary white noise process with spectral density $S_0 = 1$ m^2/s^3. The design variables are the dimensions of the cross section of the columns elements (y_i, $i = 1, \ldots, 5$). In this application, two failure modes are considered, and they are defined as follows: The first failure mode is assumed when the displacement at the top of the system relative to the ground reaches some critical level for the first time in the stationary portion of the response, and the second failure mode is assumed when the base shear response reaches some critical level for the first time in the stationary portion of the response. The threshold level is chosen to be a multiple of the standard deviation of the corresponding nominal response, that is, $\eta = \alpha\sigma_r$, where η is the threshold level, σ_r is the standard deviation of the response, and α is a normalized measure of the threshold level. A normalized threshold level $\alpha = 3$ is considered. In this context, the nominal response is taken as the response of the system when $y_i = 0.4$ m, $i = 1, \ldots, 5$. The design criteria are given by the preference functions of the structural volume of the column elements, f_1, the failure probability P_{F_1}, given by the top displacement response (first failure mode), the failure probability P_{F_2}, given by the base shear response (second failure mode), and the dimensions of the cross section of the columns elements (y_i, $i = 1, \ldots, 5$). The preference functions of these design criteria are shown in Fig. 3. Noted that the preference functions of the design variables and failure probabilities have transition zones with respect to their crisp constraints (constraints with preference one). A linearly decreasing function is used to specify the preference values for the structural cost in terms of the structural volume, with $\mu = 1$ at the minimum allowable volume and $\mu = 0$ at the maximum allowable volume. For example, the preference function of the design criterion given in terms of the failure probability can be

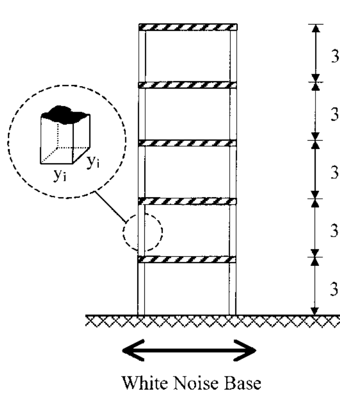

Fig. 2a Five-story shear structure.

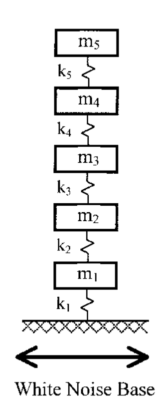

Fig. 2b Five-degree-of-freedom chainlike model.

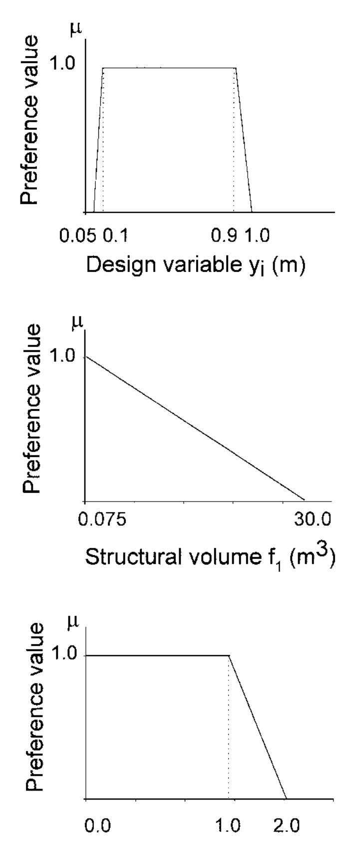

Fig. 3 Preference functions of design criteria.

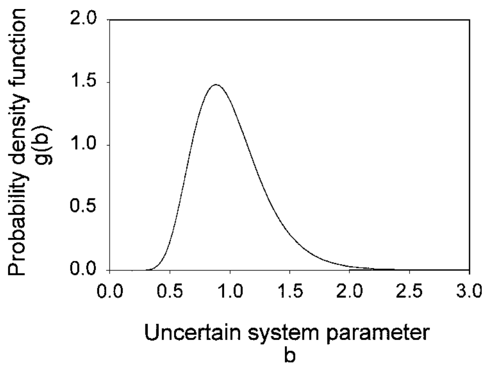

Fig. 4 Lognormal probability density function.

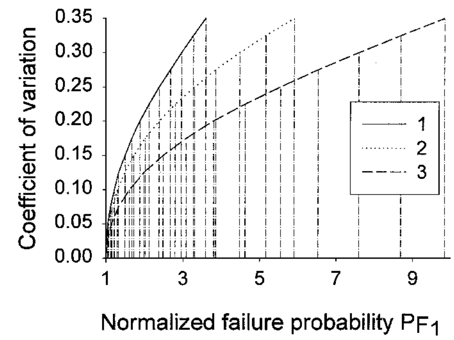

Fig. 5 Normalized failure probability P_{F_1} at the optimal conditional design as a function of the level of uncertainty of the system parameters: 1, uncertain power spectral density of the white noise excitation; 2, uncertain damping factor; and 3, uncertain modulus of elasticity.

interpreted as follows: The user prefers most of all of those values of the failure probability that are less than 1% because the preference function has its greatest possible value there. On the other hand, the user prefers least those values of the failure probability that exceed 2% because the preference function has its least possible value there. The user has selected a linear falloff between these extreme preference values for those values of the failure probability that lie between 1 and 2%. Similar interpretations have the other preference functions.

The effect of uncertainty in the system parameters on the optimal design is considered by assuming that the power spectral density of the white noise excitation S_0, the damping factor of the first two modes ζ, and the stiffness properties of the structure represented by the modulus of elasticity E are uncertain. These uncertainties are described by a lognormal distribution with most probable values given by $\bar{S}_0 = 1.0\ \mathrm{m}^2/\mathrm{s}^3$, $\bar{\zeta} = 0.05$, and $\bar{E} = 2 \times 10^9\ \mathrm{N/m}^2$. Figure 4 shows a lognormal probability density function of a random variable b with most probable value $\bar{b} = 1$, with a coefficient of variation of 30%. Recall that the coefficient of variation is the ratio between the standard deviation and the mean value of the uncertain parameter. Because of the simplicity of the example model, the total reliability (unconditional) can be evaluated directly by numerical integration. However, for comparison purposes, the importance sampling technique was also implemented. Validation calculation showed that the results obtained from both methods were almost identical for this example problem.

The example problem is now optimized using approximations based on the description given in the preceding sections. Modal energies are taken as intermediate response quantities and column elements section properties (moments of inertia) as intermediate variables. For the numerical implementation, only the first two modes were retained in the analysis of the system because it was found that the contribution of higher modes was negligible in this case. Version 4.0 of DOT[29] was used for the computational implementation of the problem. The relative change in the design variables (move limits) where the approximations are expected to yield reasonable results can be controlled automatically or directly by the users.[30] In this application, stepwise move limits of 40% were initially imposed on the intermediate design variables, and then they were gradually tightened at each stage of the design process. In all cases, convergence was obtained in less than 20 design cycles. Figure 5 shows the effect of the uncertainty in the system parameters on the probability of the first failure mode, P_{F_1}. First, an optimal design (y_i^*, $i = 1, \ldots, 5$), when the system parameters S_0, ζ, and E are fixed at their most probable values, is obtained (conditional design). Then, the system parameters are assumed to be uncertain and the failure probability P_{F_1} at the optimal design y_i^*, $i = 1, \ldots, 5$, is computed [Eq. (28)]. The failure probability is then normalized by the failure probability of the conditional design (fixed system parameters). The level of uncertainty in the uncertain parameters is given by the coefficient of variation. A range between 0 and 0.35 is considered in Fig. 5. The uncertainty in each of the parameters is considered separately in turn while holding the other parameters fixed at their most probable values. In Fig. 5, line 1 represents the case of uncertain power spectral density of the white noise excitation S_0, line 2 the case of uncertain damping factor ζ, and line 3 the case of uncertain modulus of elasticity E. As Fig. 5 shows, the uncertainty in some system parameters has a very significant influence on the reliability of the optimal design. It is clearly seen that uncertainties are important because they can change the failure probabilities by orders of magnitude. For example, a factor of more than nine is obtained in the case of a 35% coefficient of variation of the modulus of elasticity. Thus, the failure probability of the conditional optimal design increases more than nine times when the uncertainty in the stiffness characteristics of the system is considered. These results indicate that the optimal solution is very sensitive to variations of some system parameters. Also, comparing the results from Fig. 5, one can conclude that for this example model the uncertainty in the stiffness properties of the structure is much more important than the uncertainty in the damping characteristics and the input power spectral density. Similar results, regarding the effect of uncertainty in the system parameters on the optimal solution, are obtained for the second failure mode. That is, uncertainty in the system parameters can have an important effect on the reliability of the system measured by the response level of the base shear response.

The final designs for the conditional and unconditional optimization is presented in Table 1. The unconditional case corresponds to the optimal design when the uncertainty in the system parameters is considered explicitly during the design process. In this case, the overall reliability $P_{F_1(\mathrm{uncon})}$ is computed using the total probability theorem [Eq. (28)]. In Table 1, a 30% coefficient of variation is considered for the uncertain system parameters. Once again, each of the uncertain parameters is considered separately in turn while holding the other parameters fixed at their most probable values. As before, the effect of uncertainty in the system parameters on the optimal design is evident. It is seen that uncertain system parameters lead to larger structural elements. It is also seen that the design criterion given in terms of the second failure mode is inactive at the final design. Therefore, the optimal design for this example problem is controlled by the structural volume and the first failure mode (active design criteria). The correlation characteristics of uncertain system parameters can also have an important effect on the optimal design.

Table 2 shows the effect of spatial correlation of an uncertain system parameter on the optimal solution. In particular, the stiffness property of the system represented by the interstory stiffnesses k_i, $i = 1, \ldots, 5$, is modeled as a random vector with a given correlation structure. To illustrate this, two extreme cases are considered here. In the first case, the random vector is completely uncorrelated, whereas in the second case it is fully correlated. These results show that the effect of uncertainty in the stiffness properties is more significant in the fully correlated case. That is, the effect of the uncertainty in the random vector on the optimal design tends to decrease as the vector becomes uncorrelated. These results suggest that, in general, the effect of an uncertain system parameter on the optimal design is maximum when its spatial correlation is complete (fully correlated). Therefore, the fully correlated case may be taken as a conservative design if the spatial correlation characteristics of the parameter are not well known.

The relative importance of uncertain system parameters on the optimal design can also be obtained from sensitivity measures. Table 3

Table 1 Final designs threshold level: $\eta = 3\sigma_r$

	Uncertain system parameter			
Design criteria	Conditional	Unconditional S_0	Unconditional ζ	Unconditional E
Cross-sectional dimension y_1, m	0.473	0.490	0.497	0.515
Cross-sectional dimension y_2, m	0.447	0.463	0.470	0.488
Cross-sectional dimension y_3, m	0.428	0.443	0.450	0.467
Cross-sectional dimension y_4, m	0.414	0.429	0.436	0.453
Cross-sectional dimension y_5, m	0.388	0.402	0.408	0.422
Structural volume f_1, m^3	5.580	6.060	6.180	6.660
Failure probability P_{F_1}, % (first failure mode)	1.180	1.200	1.200	1.220
Failure probability P_{F_2}, % (second failure mode)	0.730	0.004	0.001	0.001
Overall preference λ	0.820	0.800	0.790	0.780

Table 2 Final designs threshold level: $\eta = 3\sigma_r$

Design criteria	Conditional	Unconditional uncorrelated	Unconditional fully correlated
Cross-sectional dimension y_1, m	0.473	0.489	0.515
Cross-sectional dimension y_2, m	0.447	0.484	0.488
Cross-sectional dimension y_3, m	0.428	0.462	0.467
Cross-sectional dimension y_4, m	0.414	0.447	0.453
Cross-sectional dimension y_5, m	0.388	0.418	0.422
Structural volume f_1, m^3	5.580	6.380	6.660
Failure probability P_{F_1}, % (first failure mode)	1.180	1.210	1.220
Failure probability P_{F_2}, % (second failure mode)	0.730	0.001	0.001
Overall preference λ	0.820	0.790	0.780

Table 3 Elasticity of design criteria threshold level: $\eta = 3\sigma_r$

Design criteria	S_0	ζ	E
Structural volume f_1	0.35	−0.35	−0.50
Failure probability P_{F_1} (first failure mode)	7.37	−7.87	−11.44
Failure probability P_{F_2} (second failure mode)	7.83	−8.42	−12.28

shows the sensitivities of the design criteria with respect to the uncertain parameters at the conditional optimal design. The sensitivities are compared by the so-called elasticities. For example, the elasticity of the design criterion defined by the structural volume f_1 with respect to S_0 is given by $(\partial f_1/\partial S_0)(S_0/f_1)$. The elasticities of the other design criteria are defined in a similar manner. The elasticities of the design criteria with respect to the power spectral density of the white noise excitation S_0 are positive, whereas they are negative with respect to the damping factor of the first two modes ζ and the stiffness properties of the system represented by the modulus of elasticity E. It follows from here that the failure probability of the first failure mode increases as S_0 increases whereas it decreases as ζ and E increase. This is reasonable because higher values of the power spectral density of the excitation will produce higher response levels. On the other hand, higher damping factors and stiffer systems will result in lower response levels. It is clear that the effect of the power spectral density of the white noise excitation and damping factors on the sensitivity of the design criteria is less significant than the effect of the stiffness properties of the system. Therefore, it is expected that the effect of uncertain stiffness properties on the optimal design is more important than the effect of uncertainty in the other system parameters considered in this example problem. It is seen that these conclusions agree with the results showed in Table 1.

As this simple yet illustrative numerical example demonstrates, the proposed methodology provides a framework in which the optimal design of structures with uncertain properties subjected to stochastic excitation can be determined. The use of approximations proves to be efficient for the numerical implementation of the method. This is particularly important for high-dimensional structural optimization problems where the evaluation of second-order statistics of the response at every single (out of a large number) iteration step may require an extensive amount of computational effort. In general, the use of approximation concepts dramatically reduces the number of exact system analyses required for convergence. Thus, the proposed methodology is expected to be useful in the optimization of large structural systems.

Conclusions

The effects of uncertain system parameters on the optimal design of systems subjected to uncertain loads has been presented. The reliability of the system is defined in terms of the probability that the magnitude of response quantities do not exceed some critical threshold level throughout the duration of the stochastic excitation. Approximate extreme response theories are used to obtain the reliability estimates. A multicriterion optimal design methodology based on preference aggregation rules is used in the formulation. The optimization problem is posed as a general nonlinear optimization problem, and it is solved using approximation concepts. The numerical results have shown that uncertainty in the model parameters may cause significant changes in the reliability of a system subjected to uncertain loads. In these situations, the errors or uncertainties in the specification of the system properties should be properly accounted for during the optimization process because, if they are not accounted for, the performance and reliability of the optimal design can be affected significantly. Therefore, under uncertain conditions, it is recommended to use the approach presented in this study (unconditional optimization) instead of the classical optimization approach (conditional optimization). In summary, the results of this study suggest that great caution must be exercised when interpreting optimal designs of systems subjected to stochastic excitation and when some system parameters are not precisely known. In such cases, the effect of uncertain system parameters should be explicitly considered during the design optimization process.

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